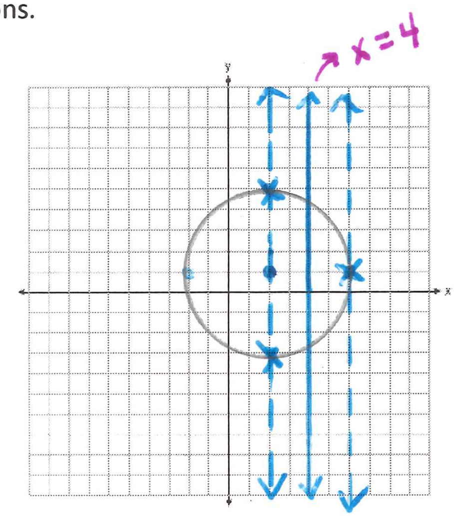
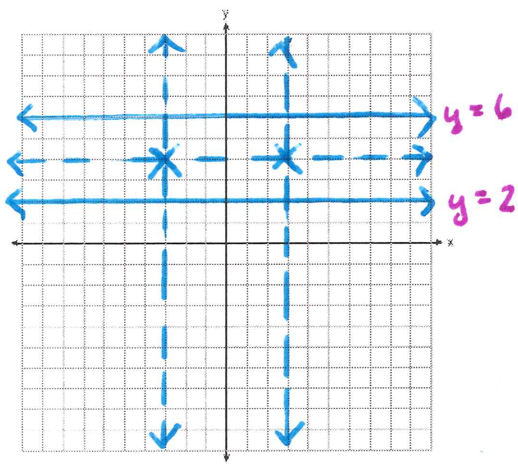
Name:_______________________________________ **Saturday Regents Review**

1) On the set of axes below, graph the locus of points that are four units from the point $(2,1)$. On the same set of axes, graph the locus of points that are two units from the line $x = 4$. State the coordinates of all points that satisfy both conditions.



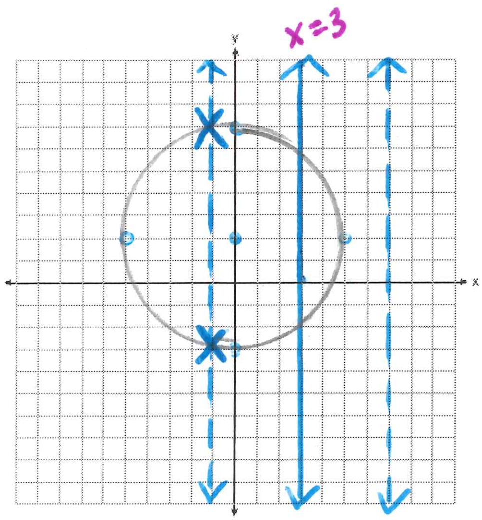
(2, -3)
(2, 5)
(6, 1)

2) On the set of coordinate axes below, graph the locus of points that are equidistant from the lines $y = 6$ and $y = 2$ and also graph the locus of points that are 3 units from the *y*-axis. State the coordinates of *all* points that satisfy *both* conditions.



(3, 4)
(-3, 4)

3) On the set of axes below, graph the locus of points that are 4 units from the line $x = 3$ and the locus of points that are 5 units from the point $(0,2)$. Label with an **X** all points that satisfy both conditions.

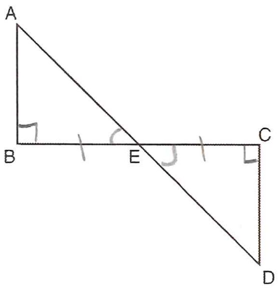


4) Given: $\overline{AD}$ bisects $\overline{BC}$ at E.

$\overline{AB} \perp \overline{BC}$

$\overline{DC} \perp \overline{BC}$

Prove: $\overline{AB} \cong \overline{DC}$



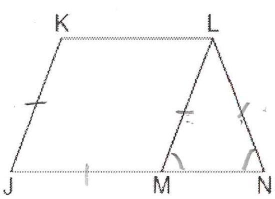
Statements	Reasons
① $\overline{AD}$ bisects $\overline{BC}$ at E	① Given
② $\overline{BE} \cong \overline{CE}$	② Def. of bisect
③ $\overline{AB} \perp \overline{BC}$, $\overline{DC} \perp \overline{BC}$	③ Given
④ $\angle B = 90°$, $\angle C = 90°$	④ Def. of $\perp$
⑤ $\angle B \cong \angle C$	⑤ Substitution
⑥ $\angle AEB \cong \angle DEC$	⑥ Vertical $\angle$s are $\cong$
⑦ $\triangle ABE \cong \triangle DCE$	⑦ ASA
⑧ $\overline{AB} \cong \overline{DC}$	⑧ CPCTC

5) Given: *JKLM* is a parallelogram.

$\overline{JM} \cong \overline{LN}$

$\angle LMN \cong \angle LNM$

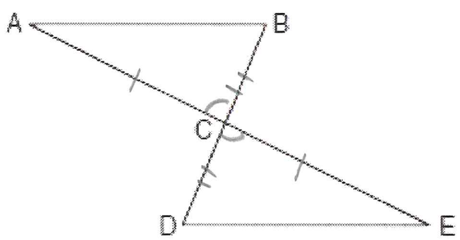
Prove: *JKLM* is a rhombus.



Statements	Reasons
① JKLM is a parallelogram	① Given
② $\overline{JK} \cong \overline{LM}$, $\overline{KL} \cong \overline{JM}$	② opp. sides in a p-gram are $\cong$
③ $\angle LMN \cong \angle LNM$	③ Given
④ $\overline{LM} \cong \overline{LN}$	④ Converse of the Isosceles $\triangle$ Theorem
⑤ $\overline{JM} \cong \overline{LN}$	⑤ Given
⑥ $\overline{JM} \cong \overline{LM}$	⑥ Transitive Property (4,5)
⑦ $\overline{KL} \cong \overline{LM}$	⑦ Transitive Property (1,6)
⑧ JKLM is a rhombus	⑧ all 4 sides are $\cong$

6) Given: $\triangle ABC$ and $\triangle EDC$, C is the midpoint of $\overline{BD}$ and $\overline{AE}$

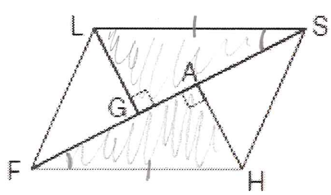
Prove: $\overline{AB} \parallel \overline{DE}$



Statements	Reasons
1. C is the midpoint of $\overline{BD}$ and $\overline{AE}$	1. Given
2. $\overline{BC} \cong \overline{DC}$, $\overline{AC} \cong \overline{EC}$	2. Def. of a midpoint
3. $\angle BCA \cong \angle DCE$	3. Vertical ∡s are ≅
4. $\triangle ABC \cong \triangle EDC$	4. SAS
5. $\angle A \cong \angle E$	5. CPCTC
6. $\overline{AB} \parallel \overline{DE}$	6. If the alt. int ∡s are ≅, then the lines are parallel.

7) Given: parallelogram *FLSH*, diagonal $\overline{FGAS}$, $\overline{LG} \perp \overline{FS}$, $\overline{HA} \perp \overline{FS}$

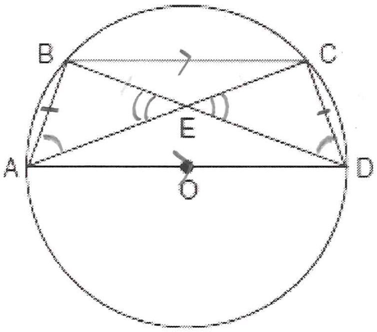
Prove: $\triangle LGS \cong \triangle HAF$



Statements	Reasons
1. Parallelogram FLSH	1. Given
2. $\overline{LS} \cong \overline{HF}$	2. opp. sides in a p-gram are ≅
3. $\overline{LS} \parallel \overline{HF}$	3. opp. sides in a p-gram are ∥
4. $\angle LSG \cong \angle HFA$	4. Alt. int. ∡s are ≅
5. $\overline{LG} \perp \overline{FS}$, $\overline{HA} \perp \overline{FS}$	5. Given
6. $\angle LGA = 90$, $\angle HAF = 90$	6. Def. of ⊥
7. $\angle LGA \cong \angle HAF$	7. Substitution
8. $\triangle LGS \cong \triangle HAF$	8. AAS

8) In the accompanying diagram of circle O, $\overline{AD}$ is a diameter with $\overline{AD}$ parallel to chord $\overline{BC}$, chords $\overline{AB}$ and $\overline{CD}$ are drawn, and chords $\overline{BD}$ and $\overline{AC}$ intersect at E.

Prove: $\overline{BE} \cong \overline{CE}$



Statements	Reasons
① $\overline{AD} \parallel \overline{BC}$	① Given
② $\overline{BA} \cong \overline{CD}$	② ∥ chord intersect ≅ arcs & the chords defined by those arcs
③ $\angle BAC \cong \angle CDB$	③ inscribed ∡s in the same arc are ≅
④ $\angle BEA \cong \angle CED$	④ vertical ∡s are ≅
⑤ $\triangle BAE \cong \triangle CDE$	⑤ AAS
⑥ $\overline{BE} \cong \overline{CE}$	⑥ CPCTC

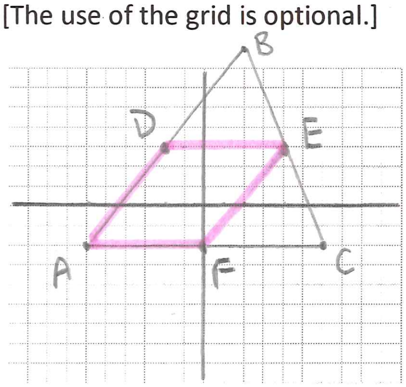
9) Given: $\triangle ABC$ with vertices $A(-6,-2)$, $B(2,8)$, and $C(6,-2)$. $\overline{AB}$ has midpoint D, $\overline{BC}$ has midpoint E, and $\overline{AC}$ has midpoint F.

Prove: *ADEF* is a parallelogram

ADEF is *not* a rhombus

ADEF is *not* a trapezoid.

[The use of the grid is optional.]



$$D = \left(\frac{-6+2}{2}, \frac{-2+8}{2}\right) = \left(\frac{-4}{2}, \frac{6}{2}\right) = (-2,3)$$

$$E = \left(\frac{2+6}{2}, \frac{8+-2}{2}\right) = \left(\frac{8}{2}, \frac{6}{2}\right) = (4,3)$$

$$F = \left(\frac{-6+6}{2}, \frac{-2+-2}{2}\right) = \left(\frac{0}{2}, \frac{-4}{2}\right) = (0,-2)$$

$$DE = \frac{3-3}{-2-4} = \frac{0}{-6} = 0 \qquad AD = \frac{-2-3}{-6--2} = \frac{-5}{-4} = \frac{5}{4}$$

$$AF = \frac{-2--2}{-6-0} = \frac{0}{-6} = 0 \qquad EF = \frac{3--2}{4-0} = \frac{5}{4}$$

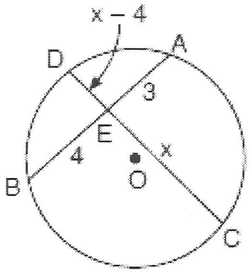
∴ ADEF is a parallelogram b/c the opposite sides are ∥. It is not a trapezoid b/c there is more than one pair of parallel sides.

$$DE = \sqrt{(-2-4)^2+(3-3)^2} = \sqrt{(-6)^2+(0)^2} = \sqrt{36+0} = \sqrt{36} = 6$$

$$AD = \sqrt{(-6--2)^2+(-2-3)^2} = \sqrt{(-4)^2+(-5)^2} = \sqrt{16+25} = \sqrt{41}$$

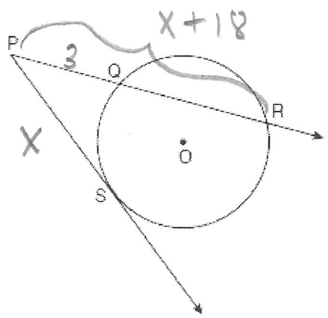
∴ ADEF is not a rhombus because all 4 sides are not ≅.

10) In the accompanying diagram of circle *O*, chords $\overline{AB}$ and $\overline{CD}$ intersect at *E*. If $AE = 3$, $EB = 4$, $CE = x$, and $ED = x - 4$, what is the value of *x*?



$$x(x-4) = 4 \cdot 3$$
$$x^2 - 4x = 12$$
$$x^2 - 4x - 12 = 0$$
$$(x-6)(x+2) = 0$$
$$x = 6 \quad \cancel{x = -2}$$

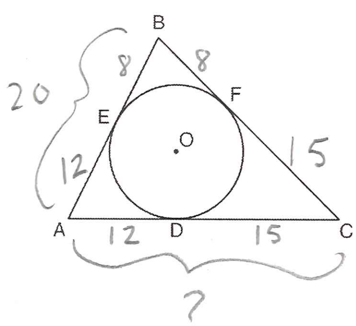
11) In the diagram below, $\overline{PS}$ is a tangent to circle *O* at point *S*, $\overline{PQR}$ is a secant, $PS = x$, $PQ = 3$, and $PR = x + 18$.



(Not drawn to scale)

$$3(x+18) = x \cdot x$$
$$3x + 54 = x^2$$
$$0 = x^2 - 3x - 54$$
$$0 = (x-9)(x+6)$$
$$x = 9 \quad \cancel{x = -6}$$

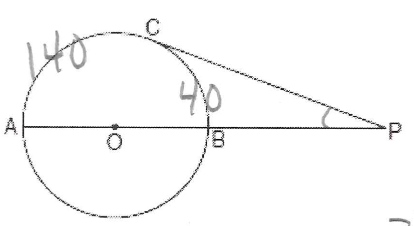
12) In the diagram below, $\triangle ABC$ is circumscribed about circle *O* and the sides of $\triangle ABC$ are tangent to the circle at points *D*, *E*, and *F*. If $AB = 20$, $AE = 12$, and $CF = 15$, what is the length of $\overline{AC}$?



$$AC = 12 + 15 = 27$$

$$\boxed{AC = 27}$$

13) In the accompanying diagram of circle *O*, diameter $\overline{AOB}$ is extended through *B* to external point *P*, tangent $\overline{PC}$ is drawn to point *C* on the circle, and $m\overset{\frown}{AC} : m\overset{\frown}{BC} = 7:2$. Find $m\angle CPA$.



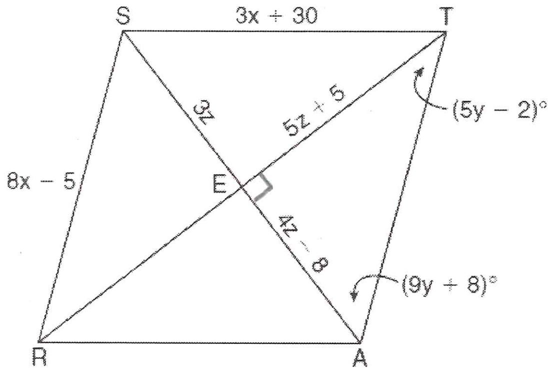
(Not drawn to scale)

$$7x + 2x = 180$$
$$9x = 180$$
$$x = 20$$

$$7(20) = 140$$
$$2(20) = 40$$

$$\angle CPA = \frac{140 - 40}{2}$$
$$\angle CPA = \frac{100}{2}$$
$$\boxed{\angle CPA = 50}$$

14) In the diagram below, quadrilateral *STAR* is a rhombus with diagonals $\overline{SA}$ and $\overline{TR}$ intersecting at *E*. $ST = 3x + 30$, $SR = 8x - 5$, $SE = 3z$, $TE = 5z + 5$, $AE = 4z - 8$, $m\angle RTA = 5y - 2$, and $m\angle TAS = 9y + 8$. Find *SR*, *RT*, and $m\angle TAS$.



$5y - 2 + 9y + 8 + 90 = 180$

$14y + 96 = 180$

$14y = 84$

$y = 6$

$\angle TAS = 9y + 8$

$= 9(6) + 8$

$= 54 + 8$

$\boxed{\angle TAS = 62}$

$8x - 5 = 3x + 30$

$5x - 5 = 30$

$5x = 35$

$x = 7$

$SR = 8x - 5$

$= 8(7) - 5$

$= 56 - 5$

$\boxed{SR = 51}$

$3z = 4z - 8$

$0 = z - 8$

$8 = z$

$RT = 5z + 5 + 5z + 5$

$RT = 10z + 10$

$RT = 10(8) + 10$

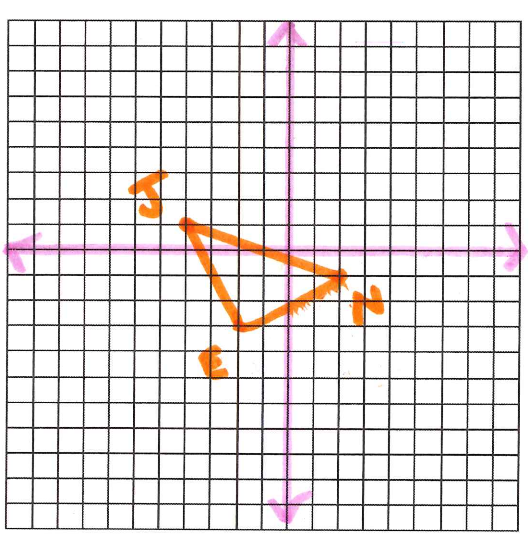
$RT = 80 + 10$

$\boxed{RT = 90}$

15) Given: $J(-4,1)$, $E(-2,-3)$, $N(2,-1)$

Prove: ΔJEN is an isosceles right triangle.

[The use of the grid is optional.]



$JE = \dfrac{-3-1}{-2--4} = \dfrac{-4}{2} = -2$

$EN = \dfrac{-1--3}{2--2} = \dfrac{2}{4} = \dfrac{1}{2}$

It is a right Δ b/c the adjacent sides have negative reciprocal slopes.

$JE = \sqrt{(-4--2)^2 + (1--3)^2}$

$= \sqrt{(-2)^2 + (4)^2}$

$= \sqrt{4 + 16}$

$JE = \sqrt{20}$

$EN = \sqrt{(-2-2)^2 + (-3--1)^2}$

$= \sqrt{(-4)^2 + (-2)^2}$

$= \sqrt{16 + 4}$

$EN = \sqrt{20}$

It is isosceles b/c 2 of the sides have the same length.

16) The angles of triangle *ABC* are in the ratio of $8:3:4$. What is the measure of the *smallest* angle?

$8x + 3x + 4x = 180$

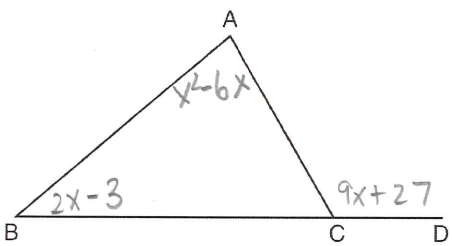
$15x = 180$

$x = 12$

$3(12) = 36$

$\boxed{\text{Smallest } \measuredangle = 36^\circ}$

17) In the diagram below of $\triangle ABC$, $\overline{BC}$ is extended to *D*. If $m\angle A = x^2 - 6x$, $m\angle B = 2x - 3$, and $m\angle ACD = 9x + 27$, what is the value of *x*?



(Not drawn to scale)

$x^2 - 6x + 2x - 3 = 9x + 27$

$x^2 - 4x - 3 = 9x + 27$

$x^2 - 13x - 30 = 0$

$(x - 15)(x + 2) = 0$

$x = 15$ $\quad$ $\cancel{x = -2}$

18) On the set of axes below, solve the following system of equations graphically and state the coordinates of *all* points in the solution.

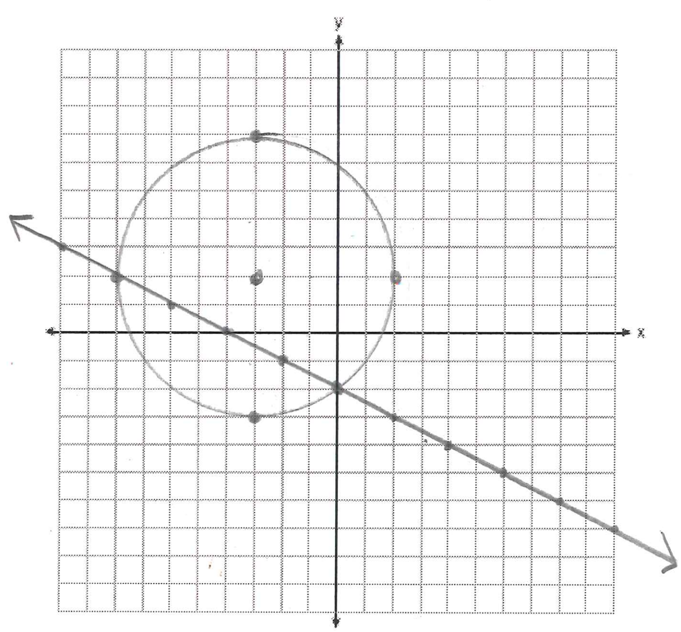
$$(x + 3)^2 + (y - 2)^2 = 25$$

$$2y + 4 = -x$$

$2y + 4 = -x$

$\frac{2y}{2} = \frac{-x}{2} - \frac{4}{2}$

$y = -\frac{1}{2}x - 2$



SOLUTION =

(0, -2)

&

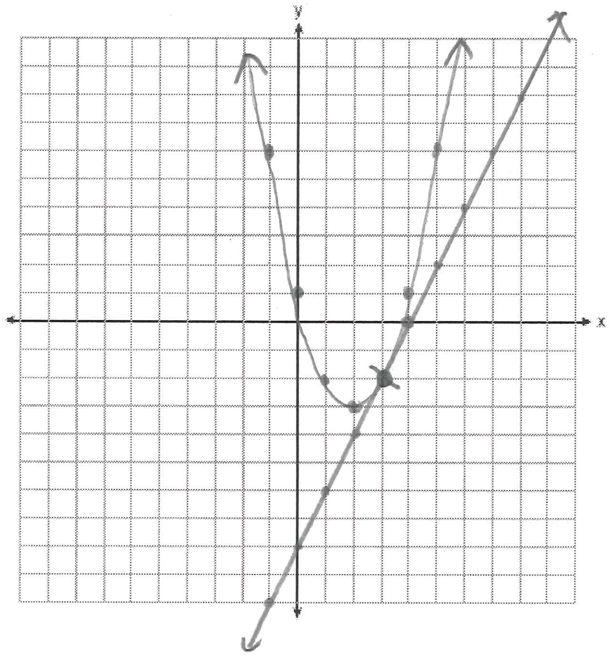
(-8, 2)

19) On the set of axes below, solve the system of equations graphically and state the coordinates of all points in the solution.

$$y = (x-2)^2 - 3$$
$$2y + 16 = 4x$$

SOLUTION =

(3, -2)

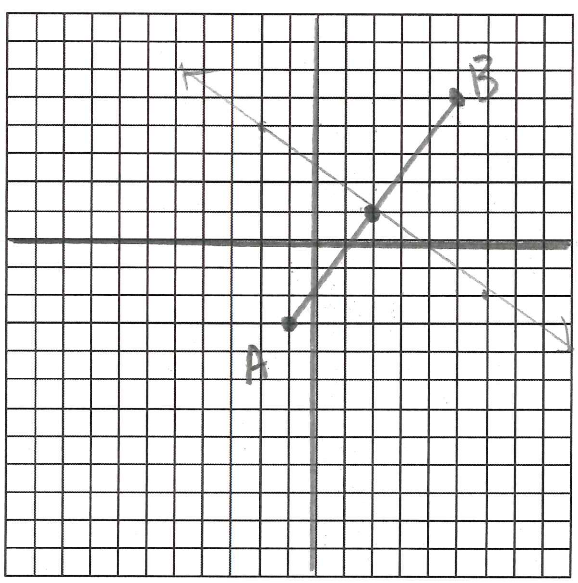


$2y + 16 = 4x$

$\frac{2y}{2} = \frac{4x - 16}{2}$

$y = 2x - 8$

20) Determine the distance between point $A(-1,-3)$ and point $B(5,5)$. Write an equation of the perpendicular bisector of $\overline{AB}$. [The use of the accompanying grid is optional.]



midpoint $= \left(\frac{-1+5}{2}, \frac{-3+5}{2}\right)$

$= \left(\frac{4}{2}, \frac{2}{2}\right) = (2,1)$

Slope AB $= \frac{5 - -3}{5 - -1} = \frac{8}{6} = \frac{4}{3}$

$\perp \rightarrow$ negative reciprocal $= -\frac{3}{4}$

$y = -\frac{3}{4}x + b$

$1 = -\frac{3}{4}(2) + b$

$1 = -1.5 + b$

$2.5 = b$

FINAL EQUATION:

$\boxed{y = -\frac{3}{4}x + 2.5}$