NAME: INFORMATION FILLED IN

Monica

Geometry Period:______

Geometry Regents (and Outcomes) Review

OUTCOME	PAGE
01: Argues with different types of reasoning in order to prove or disprove a statement	3
02: Discerns information about points, lines, and planes including parallel, perpendicular, intersecting or skew and uses appropriate notation and terminology	5
03: Uses a straightedge and a compass to make precise constructions and can argue the validity of the construction.	6 - 7
04: Be precise in calculating and applying the length and midpoint of a segment	2
05: Concludes the conditions under which a compound statement is true and can write the inverse, converse, and contrapositive of a given statement.	2
06: Graphically and algebraically discerns if lines are parallel or perpendicular on a coordinate plane and can identify the point of intersection of intersecting lines	4
07: Identifies polygons precisely and can determine angle sums and missing angle measures	4
08: Concludes if two triangles are congruent and identifies corresponding parts	8
09: Discerns and applies theorems and relationships within triangles and communicates those relationships	9
10: Discerns and applies theorems and relationships about quadrilaterals and communicates those relationships	10
11: Discerns and applies concepts of similarity in two triangles or polygons	11
12: Discerns and applies concepts of perimeter, area, surface area, and volume for two and three dimensional figures	8
13: Applies the Pythagorean Theorem and investigates relationships in special right triangles	12
14: Applies and argues properties of transformations and concepts of symmetry	14
15: Identifies parts and properties of circles and precisely determines measurements of area, circumference, arc length, angles, tangents and secants	13, 16
16: Writes, graphs, and communicates equations of circles	16
17: Graphs, solves and communicates problems using compound loci, including on a coordinate plane	15

Date of Regents: Wed. June 19th, 2013 Time: 8:30 am

Midpoint and Distance Formulas

Given two distinct endpoints of a segment on coordinate plane, (x_1, y_1) and (x_2, y_2), the midpoint of the segment can be determined by using:

$$\text{Midpoint} = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

Given two distinct endpoints of a segment on coordinate plane, (x_1, y_1) and (x_2, y_2), the length of the segment, or distance between the two points, can be determined by using:

$$\text{Distance or Length} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Compound Statements

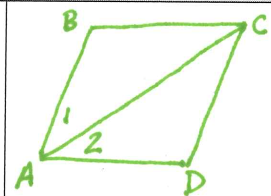
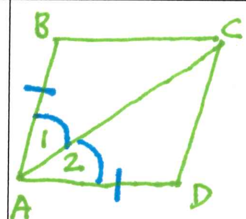
TRUE ← *TRUTH VALUE* → FALSE

Type of Compound Statement	Definition	Properties
Disjunction	A compound statement using the word "or"	True when either or both statements are true
Conjunction	A compound statement using the word "and"	True only when both statements are true
Conditional	A compound statement using "if... then"	False ONLY when the hypothesis is true and the conclusion is false.
Biconditional	Compound statement combining 2 conditionals using "if and only if"	Only true when a conditional and its converse are true

NEGATION	the opposite truth value of a statement – "not"
CONVERSE	switch the hypothesis and conclusion
INVERSE	negate the hypothesis and negate the conclusion
CONTRAPOSITIVE	switch and negate both the hypothesis and conclusion

***Remember:** A conditional and its contrapositive are always **LOGICALLY EQUIVALENT!** (They have the same truth value!)*

A PROOF is a logical argument that establishes the truth of a statement.

A proof should have the following components	*Example*
Statement of the **original problem**	 Given: Quadrilateral ABCD $\overline{AD} \cong \overline{AB}$ $\angle 1 \cong \angle 2$ Prove: $\overline{CD} \cong \overline{CB}$ } what we need to prove
Diagram, marked with the "**given**" information	 * Very important step!
Re-statement of the "given" information	STATEMENTS / REASONS 1. $\overline{AD} \cong \overline{AB}$ — 1. Given 2. $\angle 1 \cong \angle 2$ — 2. Given
Complete **supporting reasons** for each step in the proof	3. $\overline{AC} \cong \overline{AC}$ — 3. Reflexive Property 4. $\triangle ABC \cong \triangle ADC$ — 4. SAS 5. — 5.
The "**prove**" statement as the last statement	5. $\overline{CD} \cong \overline{CB}$ — 5. CPCTC

COMMONLY USED REASONS FOR PROOFS

Possible Statement	*Possible Reason*	Possible Statement	*Possible Reason*
$\angle BAC \cong \angle DAC$	Definition of a bisector	$\overline{AB} \perp \overline{CD}$	Definition of perpendicular
$\overline{AM} \cong \overline{BM}$	Definition of a bisector	$AM + BM = AB$	Segment Addition Postulate
$\angle A \cong \angle A$	Reflexive Property	$\angle 1 + \angle 2 = 90$	Definition of Complementary Angles
$\overline{AB} \cong \overline{AB}$	Reflexive Property	$\angle 1 + \angle 2 = 180$	Definition of Supplementary Angles
$\angle ACB + \angle ACD = \angle BCD$	Angle Addition Postulate	$\overline{AM} \cong \overline{BM}$	Definition of a Midpoint
$\angle 1 + \angle 2 = \angle BAD$	Angle Addition Postulate	$\angle 1 \cong \angle 3$	Substitution

Equations of Parallel and Perpendicular Lines

All linear equations can be expressed as $y = mx + b$,

where m = slope and b = y-intercept.

Parallel lines have the same slopes.	Perpendicular lines have negative reciprocal slopes.
same slope $y = \boxed{5}x + 2$ $y = \boxed{5}x - 3$	$y = \boxed{5}x + 2$ $y = \boxed{-\frac{1}{5}}x - 3$ negative reciprocals

Classifying Polygons and their angles

The sum of the interior angles of a polygon with n sides is = $(n-2) \times 180$

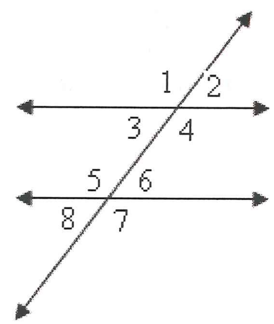
The sum of the exterior angles of a polygon with n sides is = $360°$

The measure of one interior angle of a regular polygon with n sides is = $\frac{(n-2)\times 180}{n}$

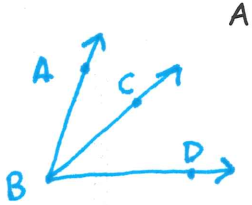
The measure of one exterior angle of a regular polygon with n sides is = $\frac{360}{n}$

n	Name of Polygon	Sum of Interior Angles	Measure of one interior angle in a regular n-gon	Measure of one exterior angle in a regular n-gon
3	triangle	180°	60°	120°
4	quadrilateral	360°	90°	90°
5	pentagon	540°	108°	72°
6	hexagon	720°	120°	60°
7	septagon/heptagon	900°	≈128.6°	≈51.4°
8	octagon	1080°	135°	45°
9	nonagon	1260°	140°	40°
10	decagon	1440°	144°	36°
12	dodecagon	1800°	150°	30°

Lines and Planes

Two Parallel Lines Cut by a Transversal	Type of Angle	Angle Pair(s)	Relationship
	Corresponding	∠1 & ∠5, ∠2 & ∠6, ∠3 & ∠8, ∠4 & ∠7	≅
	Alternate Interior	∠3 & ∠6, ∠4 & ∠5	≅
	Alternate Exterior	∠1 & ∠7, ∠2 & ∠8	≅
	Same-side Interior	∠3 & ∠5, ∠4 & ∠6	supplementary
	Same-side Exterior	∠1 & ∠8, ∠2 & ∠7	supplementary
	Vertical Angles	∠1 & ∠4, ∠2 & ∠3, ∠5 & ∠7, ∠6 & ∠8 * Note: ∥ lines not needed for these ∠s	≅

Angle Addition Postulate



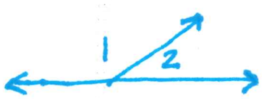
∠ABC + ∠CBD = ∠ABD

Segment Addition Postulate

* AC − AB = BC

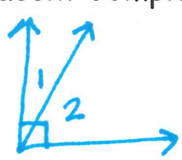
AB + BC = AC

Adjacent Supplementary Angles



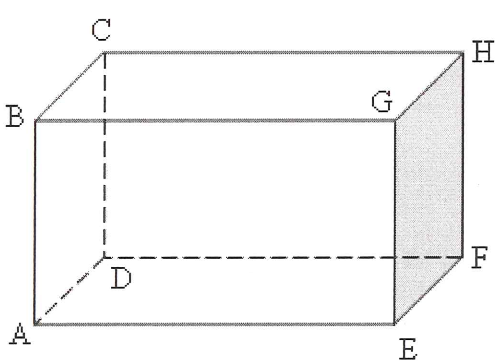
$\angle 1 + \angle 2 = 180°$

Adjacent Complementary Angles



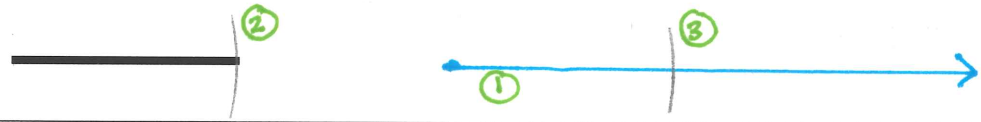
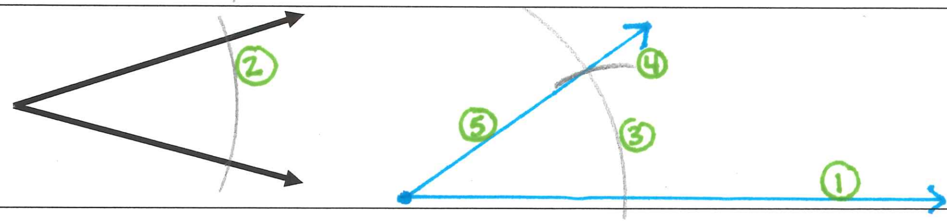
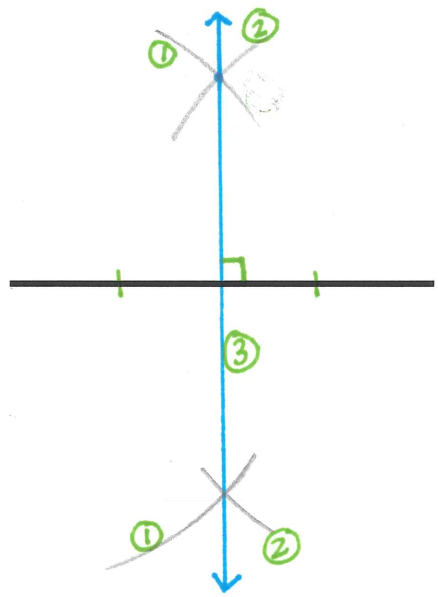
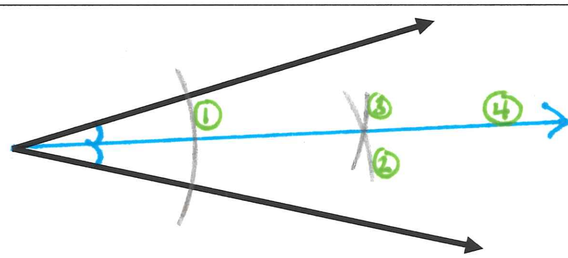
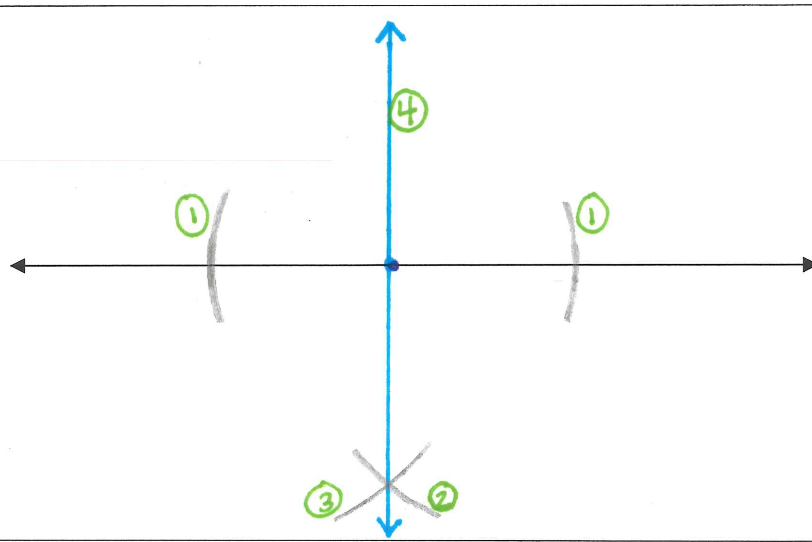
$\angle 1 + \angle 2 = 90°$

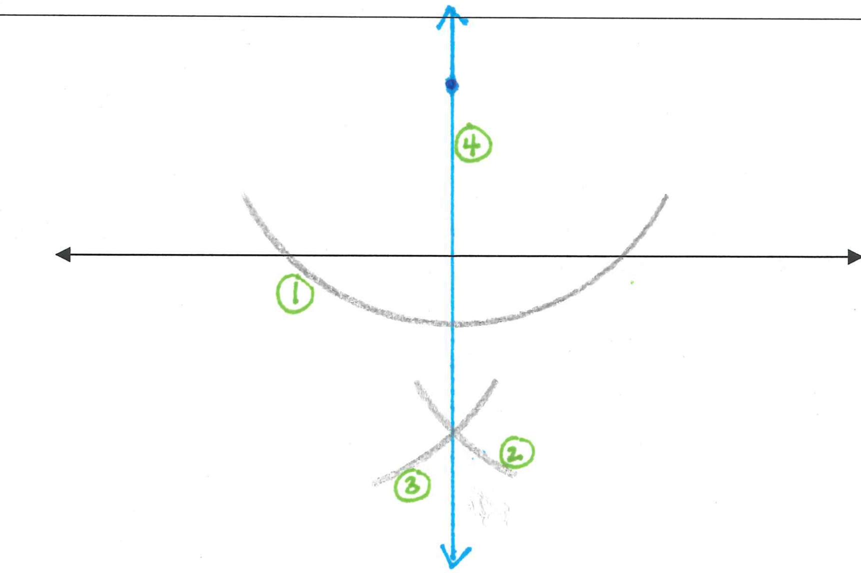
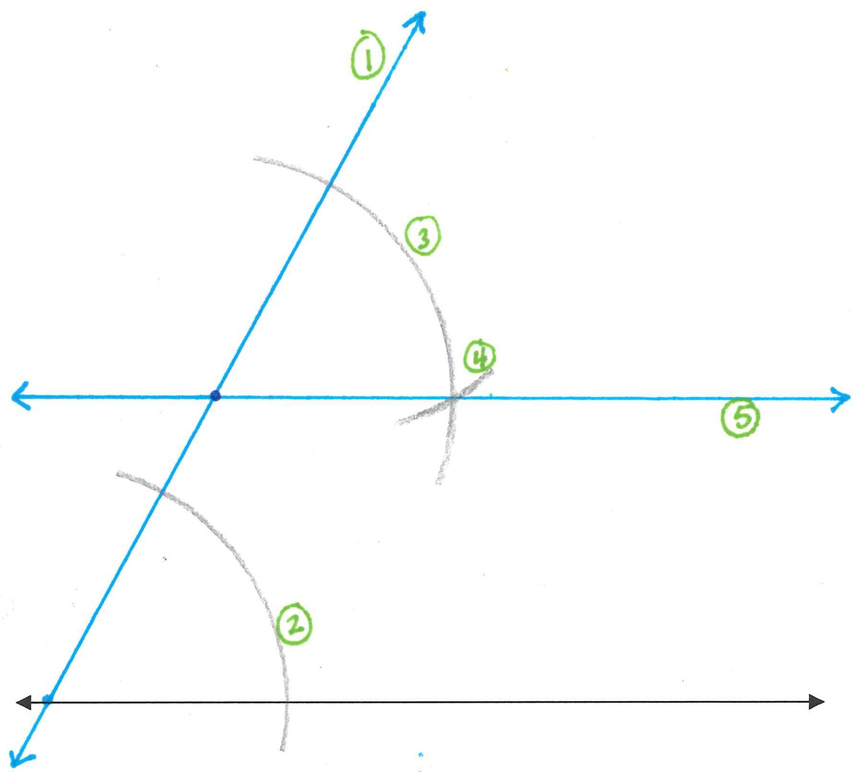
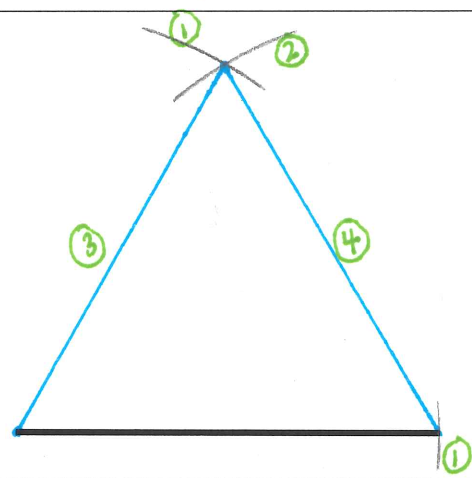
Undefined Terms		
Point	Line	Plane
A	$\overleftrightarrow{AB}$	ABC



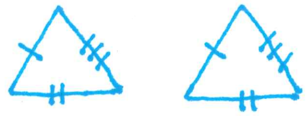
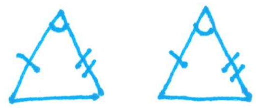
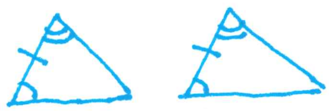
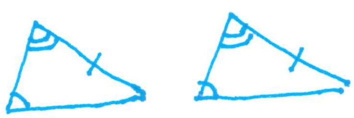
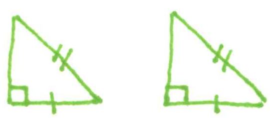
Important Terms	
parallel ∥ = never intersect (coplanar)	$\overleftrightarrow{BG} \parallel \overleftrightarrow{CH}$
perpendicular ⊥ = 90° angles (coplanar)	$\overleftrightarrow{AB} \perp \overleftrightarrow{BG}$
skew = never intersect (non-coplanar)	$\overleftrightarrow{AB}$ & $\overleftrightarrow{DF}$
collinear = points on the same line	
coplanar = points and lines on the same plane	

Constructions

Copy a Segment	
Copy an Angle	
Perpendicular Bisector	
Angle Bisector	
Perpendicular through a point on a line	

<table>
<tr><td>Perpendicular through a point off a line</td><td>
</td></tr>
<tr><td>Parallel Line through a point off a line</td><td>
</td></tr>
<tr><td>Equilateral Triangle</td><td>
</td></tr>
</table>

Congruent Triangles

POSTULATE/ THEOREM	PICTURE
SSS	
SAS	
ASA	
AAS	
HL	 * RIGHT TRIANGLES ONLY

We NEVER use SSA or ASS. (No bad words in math!)

If we know two triangles are congruent, then we can prove all of their corresponding parts are congruent. For short, we use CPCTC.

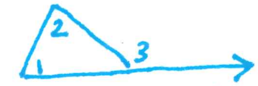
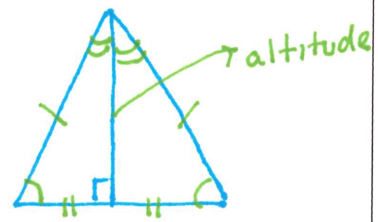
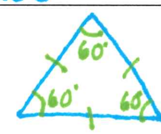
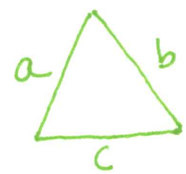
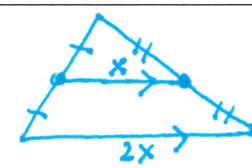
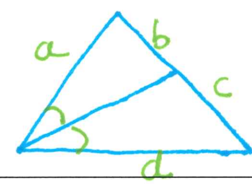
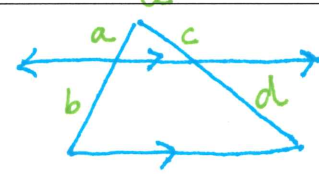
(Corresponding Parts of Congruent Triangles are Congruent)

Area, Surface Area, and Volume

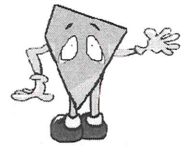
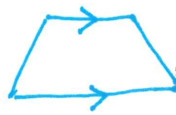
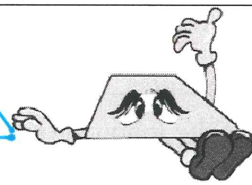
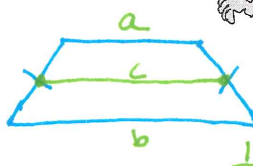
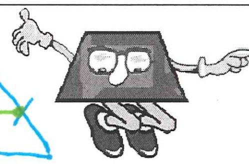
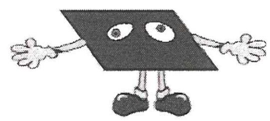
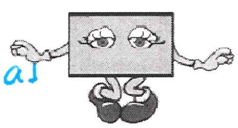
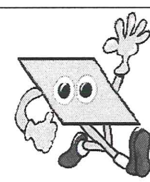
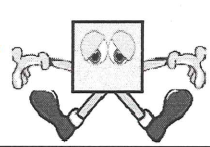
Provided Formulas	Formulas we need to know!
Volume Cylinder = Bh = $\pi r^2 h$	Area Circle = πr^2
Volume Pyramid = $\frac{1}{3}Bh$ = $\frac{1}{3}s^2h$ or $\frac{1}{3}lwh$	Circumference Circle = $2\pi r$ or πd
Volume Cone = $\frac{1}{3}Bh$ = $\frac{1}{3}\pi r^2 h$	Area Rectangle = lw
Volume Sphere = $\frac{4}{3}\pi r^3$	Volume Rectangular Prism = lwh
Lateral Area Cylinder = $2\pi rh$	Surface Area Rectangular Prism = $2lw + 2lh + 2wh$
Lateral Area Cone = πrl	Surface Area Cylinder = $2\pi r^2 + 2\pi rh$
Surface Area Sphere = $4\pi r^2$	

Properties of Triangles

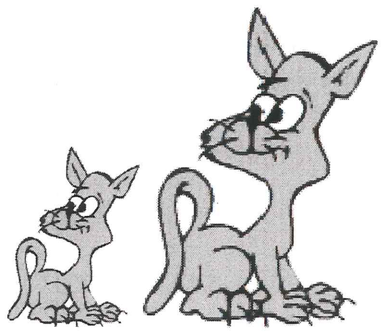
There are 180° in a Δ!

Theorem/ Property	Description
Exterior Angle Theorem	The sum of the 2 remote interior ∡s equal the exterior ∡.  $\angle 1 + \angle 2 = \angle 3$
Properties of Isosceles Triangles	1. 2 ≅ sides 2. 2 ≅ ∡s (opposite the ≅ sides) 3. The altitude to the base bisects the vertex ∡ and the base 
Equilateral Triangles	- 3 ≅ sides - each ∡ = 60° 
Inequalities in Triangles	1. The sum of any 2 sides must exceed the third side 2. The largest ∡ is opposite the largest side 3. The largest side is opposite the largest ∡.  $a+b>c$ $b+c>a$ $a+c>b$
Triangle Midsegment Theorem	The segment connecting the midpoints of the sides is parallel to the base and 1/2 of the base 
Triangle Angle-Bisector Theorem	The ∡ bisector of an ∡ cuts the opposite side to create proportional side lengths  $\frac{a}{b} = \frac{d}{c}$
Side-Splitter Theorem	If a line cuts through 2 sides of a Δ so that it's ∥ to the base, the side lengths are proportional.  $\frac{a}{b} = \frac{c}{d}$
INSIDE Points of Concurrency INSIDE Right = on Obtuse = outside Acute = inside	Perpendicular Bisectors = CIRCUMCENTER (this point is equidistant to all 3 vertices) * used to circumscribe a circle about a Δ. Angle Bisectors = INCENTER (this point is equidistant to all 3 side lengths) * used to inscribe a circle in a Δ Medians = CENTROID (cuts each median in such a way so the distance from the vertex to the centroid is double the distance from the centroid to the midpoint) Altitudes = ORTHOCENTER

Properties of Quadrilaterals

QUADRILATERAL Any 4-sided figure 	A quadrilateral is any four sided figure. Do not assume any additional properties for a quadrilateral unless you are given additional information.
TRAPEZOID A quadrilateral w/ one pair of ∥ sides  	A trapezoid has ONLY ONE set of parallel sides. When proving a figure is a trapezoid, it is necessary to prove that two sides are parallel and two sides are not parallel.
ISOSCELES TRAPEZOID - non-∥ sides are ≅ - midsegment = $\frac{1}{2}$ the sum of the bases  $\frac{1}{2}(a+b) = c$ 	Never assume that a trapezoid is isosceles unless you are given (or can prove) that information.
PARALLELOGRAM - opp. sides are ∥ - opp. sides are ≅ - diagonals bisect each other - opp. ∡s are ≅ 	Notice how the properties of a parallelogram come in sets of twos: two properties about the sides; two properties about the angles; two properties about the diagonals. Use this fact to help you remember the properties.
RECTANGLE - everything a parallelogram has PLUS - 4 right ∡s - diagonals are ≅ 	If you know the properties of a parallelogram, you only need to add 2 additional properties to describe a rectangle.
RHOMBUS - everything a parallelogram has PLUS - diagonals are perpendicular - diagonals bisect the angles - 4 ≅ sides 	A rhombus is a slanted square. It has all of the properties of a parallelogram plus three additional properties.
SQUARE - everything a parallelogram, rectangle & rhombus have 	The square is the most specific member of the quadrilateral family. It has the largest number of properties.

SIMILARITY



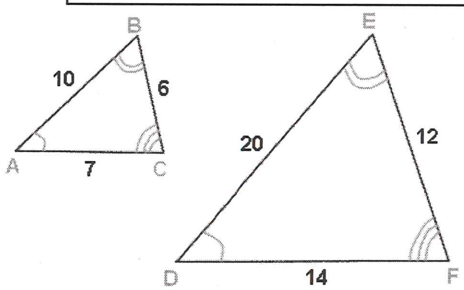
The cat on the right is an enlargement of the cat on the left. They are exactly the same shape, but they are **NOT** the same size.

These cats are **similar** figures.

SIMILARITY SYMBOL

$\sim$

SIMILAR = the same shape but different sizes

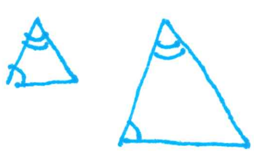
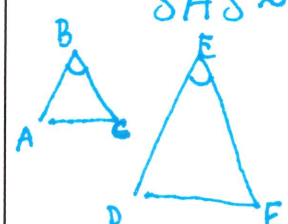
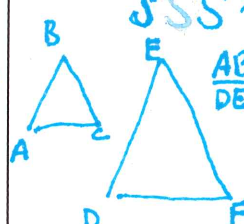


$\triangle ABC \sim \triangle DEF$

FACTS ABOUT SIMILAR TRIANGLES

$\angle A \cong \angle D$	$\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$
$\angle B \cong \angle E$	$\downarrow \quad \downarrow \quad \downarrow$
$\angle C \cong \angle F$	$\left.\frac{10}{20} = \frac{6}{12} = \frac{7}{14}\right\}$ all $= \frac{1}{2}$ which is the similarity ratio

PROVING TRIANGLES ARE SIMILAR

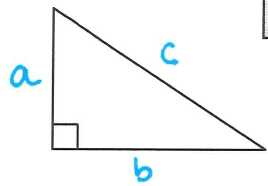
AA~	SAS~	SSS~
	 $\frac{AB}{DE} = \frac{BC}{EF}$	 $\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$

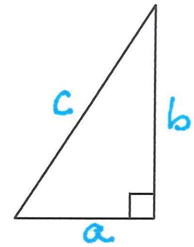
The **SIMILARITY RATIO** is the ratio of the corresponding sides of two similar figures or solids. If the similarity ratio is a:b, then...

the ratio of their perimeter (and corresponding side lengths) is $a:b$	the ratio of their areas (or surface areas) is $a^2:b^2$	the ratio of their volumes is $a^3:b^3$

REMEMBER! In similar figures, the ratio of the angle measures is always 1 : 1 ! * ∠s are ≅ !

Pythagorean Theorem

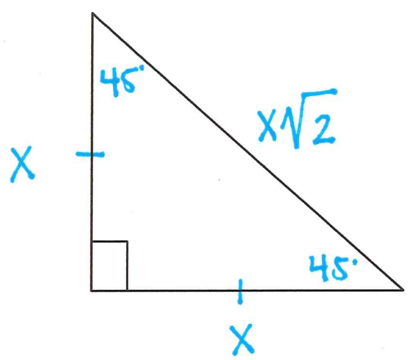


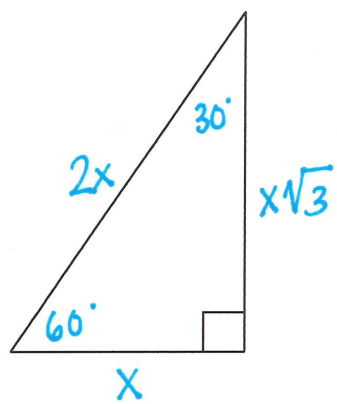


Acute Triangles	Right Triangles	Obtuse Triangles
$a^2 + b^2 > c^2$	$a^2 + b^2 = c^2$	$a^2 + b^2 < c^2$

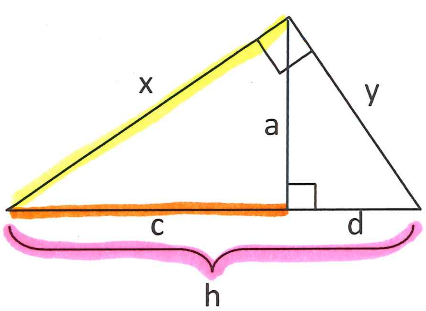
	COMMON PYTHAGOREAN TRIPLES	
3, 4, 5	5, 12, 13	8, 15, 17

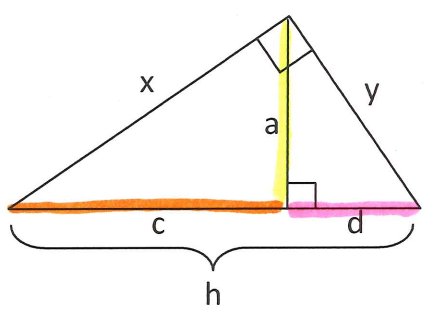
Special Right Triangles

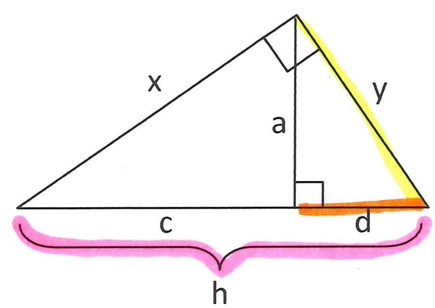




Similarity in Right Triangles





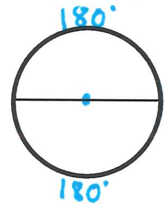


$$\frac{c}{x} = \frac{x}{h} \qquad \frac{c}{a} = \frac{a}{d} \qquad \frac{d}{y} = \frac{y}{h}$$

Circles

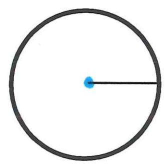
Parts and Properties of a Circle

Diameter



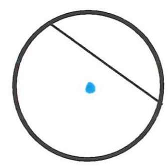
cuts the circle in half

Radius



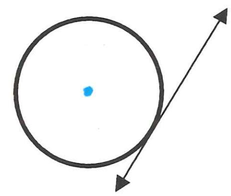
half of the diameter

Chord



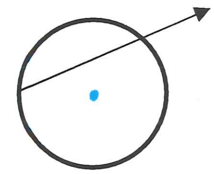
segment that joins 2 points on a circle

Tangent



a line outside of the circle that intersects in exactly 1 point

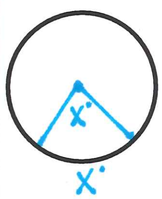
Secant



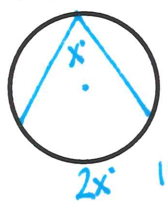
an extended chord (line or ray)

Angle and Arc Relationships *(There are 360° in a circle!)*

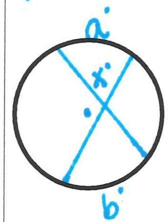
Central ∡ = intercepted arc



Inscribed ∡ = $\frac{1}{2}$ of the intercepted arc

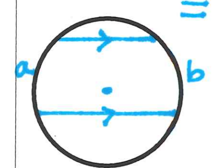


∡ formed by 2 chords



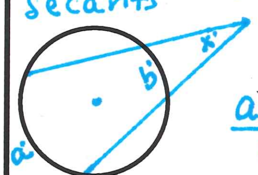
$x = \frac{a+b}{2}$

∥ chords intercept ≅ arcs



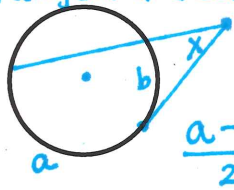
$a = b$

∡ formed by 2 secants



$\frac{a-b}{2} = x$

∡ formed by tangent & secant

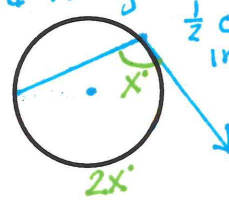


$\frac{a-b}{2} = x$

∡ formed by 2 tangents

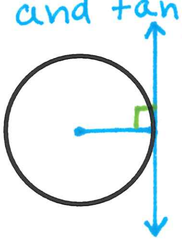
$\frac{a-b}{2} = x$

∡ formed by chord & tangent = $\frac{1}{2}$ of intercepted arc

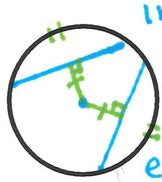


Length Relationships

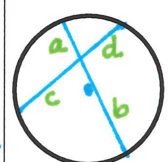
Radius (or diameter) and tangent are ⊥



≅ chords intercept ≅ arcs and are equidistant from center

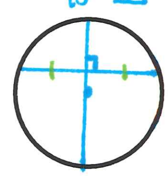


intersecting chords

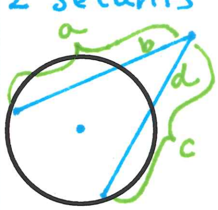


$a \cdot b = c \cdot d$

A diameter that is ⊥ to a chord, bisects the chord

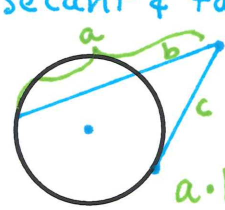


2 secants



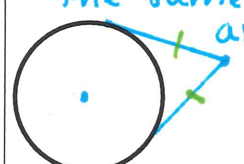
$a \cdot b = c \cdot d$

secant & tangent

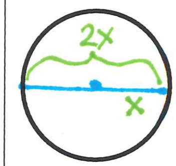


$a \cdot b = c^2$

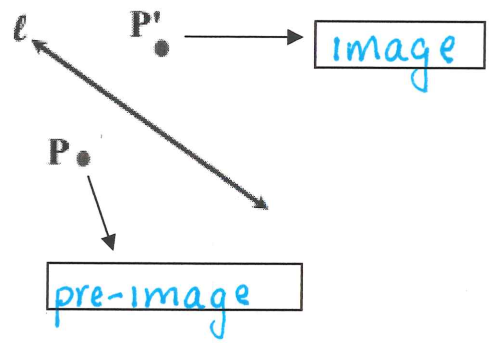
2 tangents from the same point are ≅



Radius is half of the diameter

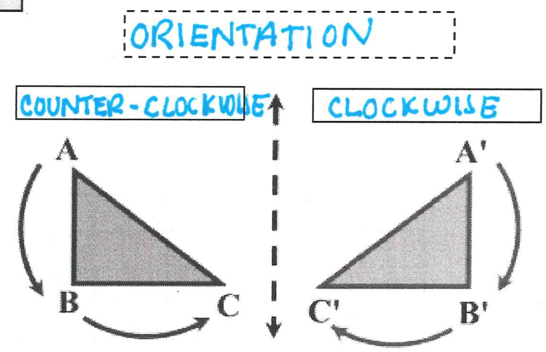


Transformations



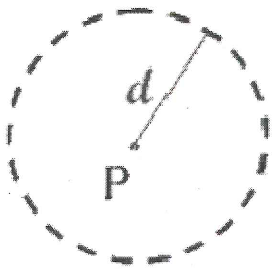
ISOMETRY

A transformation that preserves length

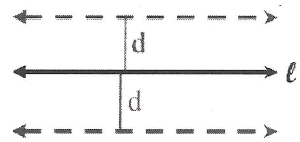


Name of Transformations	Properties	Example	What is preserved?	Is it an isometry? (Direct/Opposite)
Translation	slides an object a set distance in a given direction	$T_{2,-3}(x,y) =$ $(x,y) \to (x+2, y-3)$	•length •orientation •∡ measures	Yes! (Direct)
Reflection	flips an object over a point or line	$r_{y\text{-axis}}(x,y) =$ $(x,y) \to (-x,y)$ $r_{y=x}(x,y) =$ $(x,y) \to (y,x)$	•length •∡ measures	Yes! (opposite)
Rotation	turns an object a set # of degrees	$R_{90°}(x,y) =$ $(x,y) \to (-y,x)$	•length •∡ measures •orientation	Yes! (Direct)
Dilation	enlarges or shrinks an object by a set #	$D_2(x,y) \to$ $(x,y) \to (2x, 2y)$	•∡ measures •orientation	No!

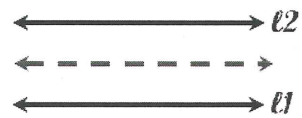
LOCUS



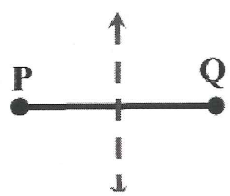
The locus of points at a fixed distance, ***d***, from point ***P*** is a circle with the given point ***P***as its center and ***d*** as its radius.



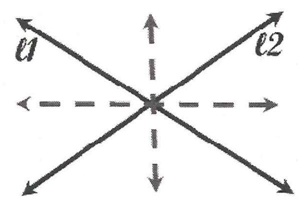
The locus of points at a fixed distance, ***d***, from a line, ℓ, is a pair of parallel lines ***d***distance from $\ell$and on either side of ℓ.



The locus of points equidistant from two points,***P*** and ***Q***, is the perpendicular bisector of the line segment determined by the two points.



The locus of points equidistant from two parallel lines, ℓ_1 and ℓ_2 , is a line parallel to both ℓ_1 and $\ell_2$and midway between them.



The locus of points equidistant from two intersecting lines, ℓ_1 and ℓ_2, is a pair of bisectors that bisect the angles formed by ℓ_1 and ℓ_2 .

Steps for Solving Locus Problems

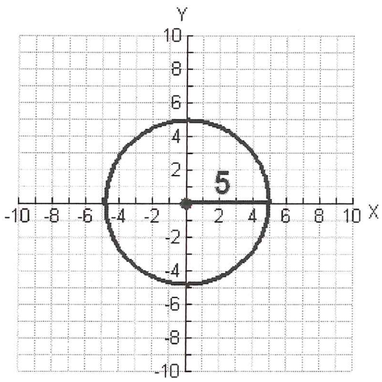
1. Draw a diagram showing the given lines and points
2. Read carefully to determine the needed condition(s).
3. Locate 1 point that satisfies the needed condition and plot it on your diagram. Repeat this process until you notice a pattern.
4. Connect your points with a dashed line to indicate the locus.
5. Describe the locus in words (circle, ll lines, etc.)
6. If 2 conditions exist, repeat the steps on the same diagram and identify the points of intersection.

Equations of Circles

Circle with Center at Origin (0,0)

$x^2 + y^2 = r^2$

where the center is (0,0) and the radius is r.

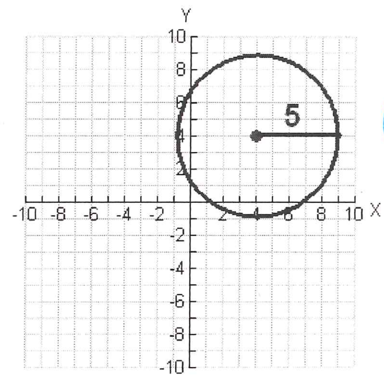


$x^2 + y^2 = 25$

Circle with Center at Point (h,k)

$(x-h)^2 + (y-k)^2 = r^2$

where the center is (h,k) and the radius is r



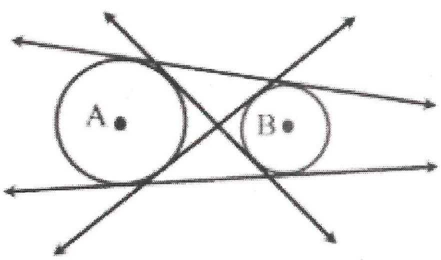
$(x-4)^2 + (y-4)^2 = 25$

Common Tangents

Common tangents are lines or segments that are tangent to more than one circle at the same time.

4 Common Tangents

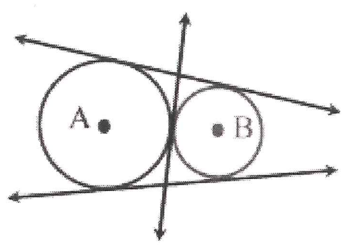
(2 completely separate circles)



2 external tangents
2 internal tangents

3 Common Tangents

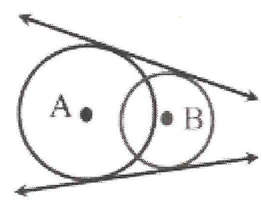
(2 externally tangent circles)



2 external tangents
1 internal tangent

2 Common Tangents

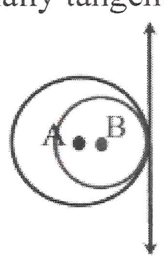
(2 overlapping circles)



2 external tangents
0 internal tangents

1 Common Tangent

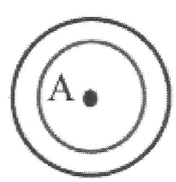
(2 internally tangent circles)



1 external tangent
0 internal tangents

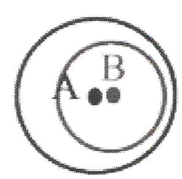
0 Common Tangents

(2 concentric circles)
Concentric circles are circles with the same center.



0 external tangents
0 internal tangents

(one circle floating inside the other, without touching)



0 external tangents
0 internal tangents